Model Checking Vs. Generalized Model Checking: Semantic Minimizations for Temporal Logics

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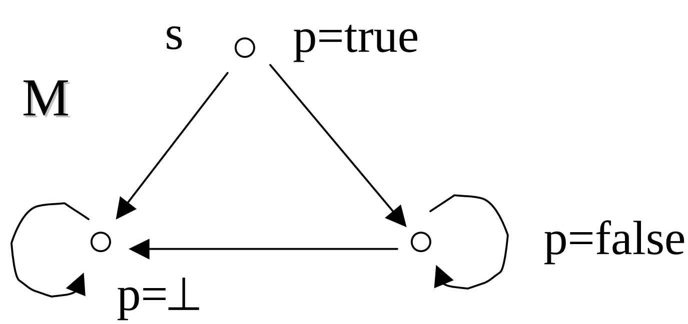
Verification via Automatic Abstraction

- Implemented in software model checkers like SLAM, BLAST,…
- Traditional iterative abstraction procedure:
  1. Abstract: generate a finite abstraction A from the concrete program C such that A simulates C (using predicate abstraction, theorem proving)
  2. Check: given any universal temporal-logic formula ϕ, compute $[A \models \phi]$:
     if $[A \models \phi]$ = true, then return true (we then know $[C \models \phi]$ = true)
  3. Refine: (checking of A is inconclusive) refine A, then go to Step 1
     - Ex: with predicate abstraction, add predicates to refine the model
- Limitations:
  - Restricted to universal properties (no existential properties)
  - $[A \models \phi]$ = false does not imply anything about C
  - Could the analysis be more precise for an acceptable cost?

A Solution: 3-Valued Models and Logics

- Richer models A that distinguish what is true/false/unknown of C
  - Example: "partial Kripke structure" [Fitting92, Bruns-G99]



  - Other example: "Modal Transition System" (may/must trans.) [Larsen+88]
  - These formalisms are all equally expressive [G-Jagadeesan03]

- Reasoning about 3-valued models requires 3-valued temporal logic
  - Ex: $[(M,s) \models p] = \text{true}$, $[(M,s) \models AXp] = \text{false}$, $[(M,s) \models EXp] = \perp$

- Complexity of 3-valued MC = complexity of MC [Bruns-G00]

New Abstract-Check-Refine Process

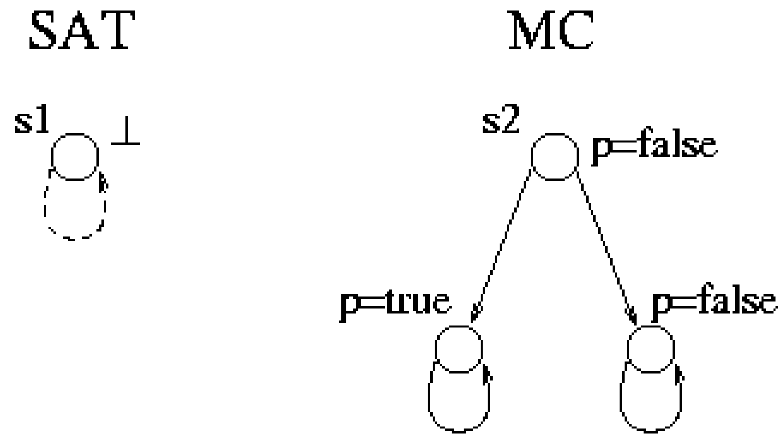
- New procedure for automatic abstraction: (3 improvements)

  [G-Jagadeesan02, G-Huth-Jagadeesan01,…]

  1. Abstract: generate a 3-valued abstraction A from the concrete program C that preserves *true*, *false*, *unknown* properties of C (same cost)
     (Formally, $A \preceq C$ where $\preceq$ is the abstraction preorder on 3-valued models)
  2. Check: given any temporal-logic formula ϕ,
     - (3-valued model checking) compute $[A \models \phi]$: (same cost)
       if $[A \models \phi]$ = true or false, then return true or false (respectively)
     - (generalized model checking)
       if there is no concretization C of A such that C satisfies ϕ, return false
       if there is no concretization C of A such that C violates ϕ, return true
  3. Refine: (checking of A is inconclusive) refine A, then go to Step 1

Generalized Model Checking (GMC)

- Definition: [Bruns-G00]
  Given a program abstraction A and a temporal logic formula ϕ, does there exist a concretization C of A such that C satisfies ϕ?



- GMC is a generalization of both
  - Satisfiability (SAT)
  - Model Checking (MC)

- GMC can be more expensive than MC (since it includes SAT)
  - in | ϕ | (but worst-case and ϕ is usually short) [Bruns-G00]
  - in |A| (quadratic) but linear for persistence (incl. safety) properties [G-J02]

- GMC can also be more precise than MC…

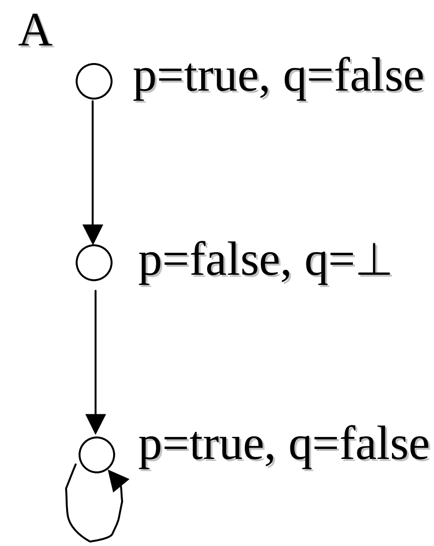
Example where GMC is more precise than MC

```
Program P( ) {
  int x,y = 1,0;
  x,y = 2*f(x), f(y);
  x,y = 1,0;
}
```

Predicate abstraction $\longrightarrow$

p: “x is odd”

q: “y is odd”



Property “(eventually y is odd) and (always, x is odd or y is even)”

is represented by the LTL formula $\phi = F(q) \wedge G(p \vee \neg q)$

$MC(A,\phi) = \perp$ …but $GMC(A,\phi) = false!$

How often is GMC more precise than MC?

- Motivation for this paper!
- More generally, how to reduce GMC(A,ϕ) to MC(A,ϕ')? (independently of A)
- ϕ' is called a semantic minimization of ϕ; problem already studied for Propositional Logic (PL) [Blamey80, Reps-Loginov-Sagiv02]
- **Theorem**: [Blamey80] for all $\phi \in$ PL, there is a sem min $\phi' \in$ PL
- Note: $|\phi'|$ can be much larger than $|\phi|$! (since computing ϕ' is as hard as GMC, which is as hard as SAT, hence NP-hard for PL)
- What about temporal logics?

Semantic Minimizations for PML and μL

- Propositional Modal Logic (PML): $\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid EX\ \phi$
- **Theorem:** For all $\phi \in$ PML, there is a sem min $\phi' \in$ PML
- Proof idea:
  - build a tree automaton $A^3(\phi)$ that accepts a 3-valued labeled tree T^3 *iff* there exists a 2-valued tree T such that $T^3 \preceq T$ and T satisfies ϕ
  - Translate $A^3(\phi)$ back into a PML formula ϕ' of modal depth $O(|\phi|)$ (possible because $A^3(\phi)$ cannot distinguish trees at depths greater than $|\phi|$)
- Modal mu-calculus (μL): $\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid EX\ \phi \mid Z \mid \mu Z.\phi$
- **Theorem:** For all $\phi \in \mu$L, there is a sem min $\phi' \in \mu$L
- Proof idea: similar as above

Semantic Minimizations for CTL, LTL and CTL*

- For CTL, LTL and CTL*, GMC is PTIME-hard in |A| [G-03]…
- …while MC is known to be NLOGSPACE-complete in |A|
- … therefore, GMC(A,ϕ) cannot be reduced to MC(A,ϕ’) unless NLOGSPACE = PTIME !
- **Theorem:**
  The sem min of the CTL formula $\phi = A[(EX\ q_1)U(q1 \rightarrow q_2)]$ is the μL formula $\phi' = \mu Z_1.(q_1 \rightarrow q_2) \vee [\mu Z_2.AX\ Z_1 \wedge EX(q_1 \wedge (q_2 \vee Z_2))]$, which is not expressible in CTL*
- Proof idea: ϕ’ is obtained with previous construction of $A^3(\phi)$; CTL* cannot express unbounded alternation (and/or graph reach.)

Semantic Minimizations: Summary

- PL, PML and μL are closed under semantic minimizations
- while CTL, LTL and CTL* are <u>not</u>
- But for all $\phi \in$ CTL* (thus CTL,LTL), there is a sem min $\phi' \in \mu$L
- Note on first-order logic over binary relations (FOL):
  since SAT is undecidable, so is GMC, while MC is decidable.
  Thus, sem min cannot exist for all formulas of FOL.

Self-Minimization

- When do we have $\phi' = \phi$?   Such ϕ are called self-minimizing
- For any self-minimizing formula ϕ, $GMC(A,\phi) = MC(A,\phi)$ that is, GMC and MC have the same precision
- Checking for self-minimization semantically:
  - Compare the automaton $A^3(\phi)$ with an automaton $A^3(|= \phi)$ that accepts exactly all 3-valued labeled trees T^3 such that $[T^3 |= \phi] = true$
  - By construction, $L(A^3(\phi)) \subseteq L(A^3(|= \phi))$
  - If $L(A^3(|= \phi)) \subseteq L(A^3(\phi))$, then ϕ is self-minimizing
  - If $\phi \in \mu L$, these automata are parity tree automata and $A^3(\phi)$ can be of size exponential in $|\phi|$
  - Thus, such a semantic (automata-based) check is precise but expensive!

Syntactic Tests for Self-Minimization

- Checking for self-minimization syntactically:
  - linear in $|\phi|$ but incomplete (less precise than semantic check)

- Example of sufficient condition: (*)
  Any formula that does not contain any atomic proposition in mixed polarity (in its negation normal form) is self-minimizing.

- Many frequently-used formulas satisfy this condition:
  - (absence) $AG(q \rightarrow AG(\neg p))$, (universality) $AG(q \rightarrow AG\ p)$
  - (existence) $EF\ p$, (response) $AG(p \rightarrow AF\ q)$, etc.

- (*) is not necessary: $(\neg q_1 \vee q_2) \wedge (\neg q_2 \vee q_1)$ is self-minimizing

Temporal Patterns of Self-Minimization

- In the paper, we present grammars to identify syntactically self-minimizing formulas

  (details omitted here)

$$
\begin{aligned}
\mathrm{ps} \quad ::= \quad & \mathcal{M} \mid \mathcal{R} \mid \neg \mathrm{os} \mid \mathrm{ps} \wedge \mathrm{ps} \mid \mathrm{ps}_{\forall\#} \vee \mathrm{ps}_{\forall\#} \\
& \mathsf{EX}\mathrm{ps} \mid \mathsf{AX}\mathrm{ps} \mid \mathsf{EG}\mathrm{ps} \mid \mathsf{AG}\mathrm{ps} \\
& \mathsf{AF}\mathrm{ps}_{\forall} \mid \mathsf{A}[\mathrm{ps}_{\forall\#}\mathsf{U}\mathrm{ps}_{\forall\#}] \\
\mathrm{os} \quad ::= \quad & \mathcal{M} \mid \mathcal{R} \mid \neg \mathrm{ps} \mid \mathrm{os} \vee \mathrm{os} \mid \mathrm{os}_{\exists\#} \wedge \mathrm{os}_{\exists\#} \\
& \mathsf{EX}\mathrm{os} \mid \mathsf{AX}\mathrm{os} \mid \mathsf{EF}\mathrm{os} \mid \mathsf{AF}\mathrm{os} \\
& \mathsf{EG}\mathrm{os}_{\exists} \mid \mathsf{E}[\mathrm{os}_{\exists}\mathsf{U}\mathrm{os}] \mid \mathit{ref}(\mathit{OS})
\end{aligned}
$$

Figure 1. ps (os) **generates pessimistically (optimistically) self-minimizing formulas (resp.);** $\mathcal{M}$ **ranges over monotone formulas of** μL, $\mathcal{R}$ **over formulas in (8);** $\#$ **and** $\forall$ ($\exists$) **are as in Definition 3(5);** OS **ranges over finite subsets of** os**; and** $\mathit{ref}(\cdot)$ **is as in Definition 4.**

- Related work: study of temporal logics and normal forms for which satisfiability is efficiently decidable
  - [Emerson-Evangelist-Srinivasan90, Janin-Walukiewicz95, Demri-Schnoebelen99, Henzinger-Kupferman-Majumdar03, etc.]

Conclusions

- Study of precision of MC vs. GMC for verification via abstraction
- More generally, study of how to reduce GMC(A,ϕ) to MC (A,ϕ')
- ϕ' is called a semantic minimization of ϕ
- Like PL, PML and μL are closed under semantic minimizations, but CTL, LTL and CTL* are not
- Checking for self-minimizing formulas:
  - semantically (precise but expensive automata-based algorithms)
  - syntactically (sufficient conditions only, linear in | ϕ |)
- Good news: in practice, many formulas are self-minimizing, and MC is as precise as GMC for those